Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)
Lecture (9)
Transient Analysis (P1)

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Reference
Chapter 16

Schaum's Outline Of Theory And Problems Of Electric Circuits

https://archive.org/details/TheoryAndProblemsOfElectricCircuits

Circuits Transient Response

- When a circuit is switched from one condition to another either by a change in the applied voltage or a change in one of the circuit elements, there is a transitional period during which the branch currents and voltage drops change from their former values to new ones

- After this transition interval called the transient, the circuit is said to be in the steady state.

- Transient analysis: study of circuit behavior in transition phase.

- The steady state values can be determined using circuit laws and complex number theory.
- The transient is more difficult as it involves differential equations.

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + a_0 x(t) = f(t)$$

- General solution to the differential equation:

$$x(t) = x_p(t) + x_c(t)$$

$x_p(t)$

- Particular integral solution (or forced response particular to a given source/excitation)
- Represent the steady-state solution which is the solution to the above non-homogeneous equation

$x_c(t)$

- Complementary solution (or natural response)
- Represent the transient part of the solution, which is the solution of the next homogeneous equation:

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + a_0 x(t) = 0$$

First-Order and Second-Order Circuits

- » First-order circuits contain only a single capacitor or inductor
- » Second-order circuits contain both a capacitor and an inductor

Differential equations Solutions

- ➢ Two techniques for transient analysis that we will learn:
  - ✓ Differential equation approach.
  - ✓ Laplace Transform approach.

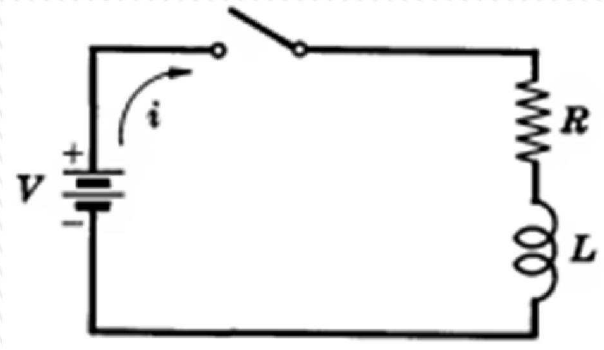
➢ Laplace transform method is a much simpler method for transient analysis

1^{st} Order R-L

DC

First-Order RL Transient Step-Response

- The switch "S" is closed at t = 0
- Apply KVL to the circuit in figure:



$$Ri + L\frac{di}{dt} = V$$

- Rearranging and using "D" operator notation :

$$\left(D + \frac{R}{L}\right)i = \frac{V}{L}$$

This Equation is a first order, linear differential equation

1. Complementary (Transient) Solution

The auxiliary equation is : $m + \frac{R}{L} = 0$

$$i = Ae^{mt} = Ae^{\frac{-R}{L}t}$$

$\tau = \frac{R}{L}$ Time constant

2. Particular (Steady-State) Solution

The steady-state value of the current for DC source is : $I_{ss} = \frac{V}{R}$

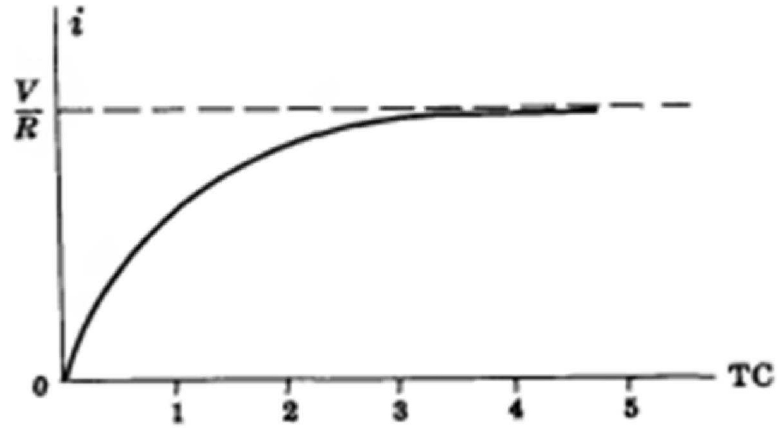
First-Order RL Transient Step-Response

- The total solution is:

$$i = Ae^{\frac{-R}{L}t} + \frac{V}{R}$$

Since The initial current is zero:

$$0 = A + \frac{V}{R}$$



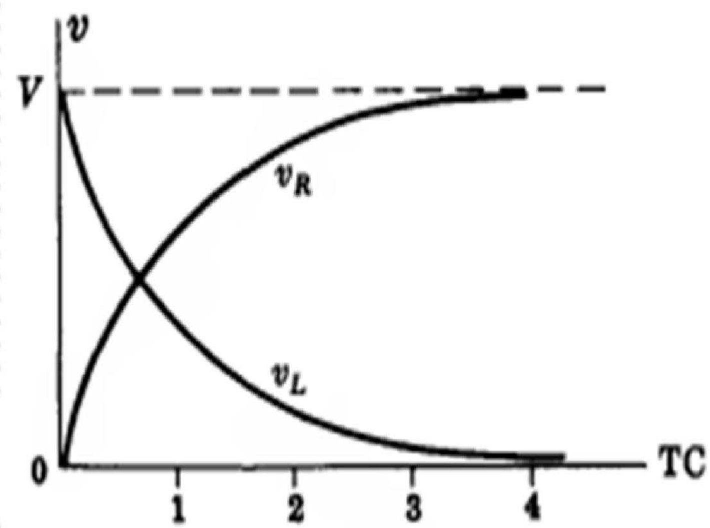
$$i = -\frac{V}{R}e^{-(R/L)t} + \frac{V}{R} = \frac{V}{R}(1 - e^{-(R/L)t})$$

- The voltage across the resistor is:

$$v_R = Ri = V(1 - e^{-(R/L)t})$$

- The voltage across the inductor is:

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left\{\frac{V}{R}(1 - e^{-(R/L)t})\right\} = Ve^{-(R/L)t}$$



$$v_R + v_L = V(1 - e^{-(R/L)t}) + Ve^{-(R/L)t} = V$$

1^{st} Order R-L

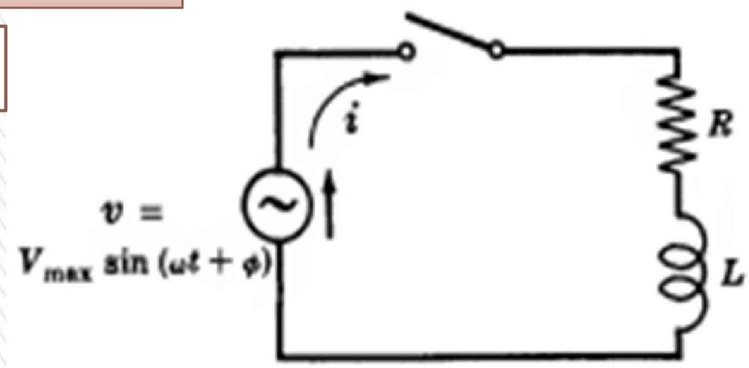
AC

Alternating Current Transients

RL Sinusoidal Transient

$$Ri + L\frac{di}{dt} = V_{max}\sin(\omega t + \phi)$$

$$\left(D + \frac{R}{L}\right)i = \frac{V_{max}}{L}\sin(\omega t + \phi)$$



1. Complementary (Transient) Solution is the solution of the homogeneous 1st order DE

The same as before, The auxiliary equation is :

$$m + \frac{R}{L} = 0$$

The complementary function is $i_c = ce^{-(R/L)t}$

2. Particular (Steady-State) Solution

The steady-state value of the current for ac source is :

$$I_{ss} = \frac{V_{max}}{\sqrt{X_L^2 + R^2}} Sin(wt + \phi - \tan^{-1}(\omega L / R))$$

Alternating Current Transients

RL Sinusoidal Transient

The complete solution is

$$i = i_c + i_p = ce^{-(R/L)t} + \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \tan^{-1} \omega L/R)$$

Use the initial condition to find the value of c

$$i_0 = 0 = c(1) + \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi - \tan^{-1} \omega L/R)$$

$$c = \frac{-V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi - \tan^{-1} \omega L/R)$$

Substituting by the constant values, we get:

$$i = e^{-(R/L)t}\left[\frac{-V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi - \tan^{-1} \omega L/R)\right] + \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \tan^{-1} \omega L/R) \quad (5$$

Examples

Examples

A series RL circuit with $R = 50$ ohms and $L = 10$ h has a constant voltage $V = 100$ v applied at $t = 0$ by the closing of a switch. Find (*a*) the equations for i, v_R and v_L, (*b*) the current at $t = .5$ seconds and (*c*) the time at which $v_R = v_L$.

(*a*) The differential equation for the given circuit is

$$50i + 10\frac{di}{dt} = 100 \quad \text{or} \quad (D+5)i = 10$$

the complete solution is $i = i_c + i_p = ce^{-5t} + 2$

At $t = 0$, $i_0 = 0$ and $0 = c(1) + 2$ or $c = -2$. Then

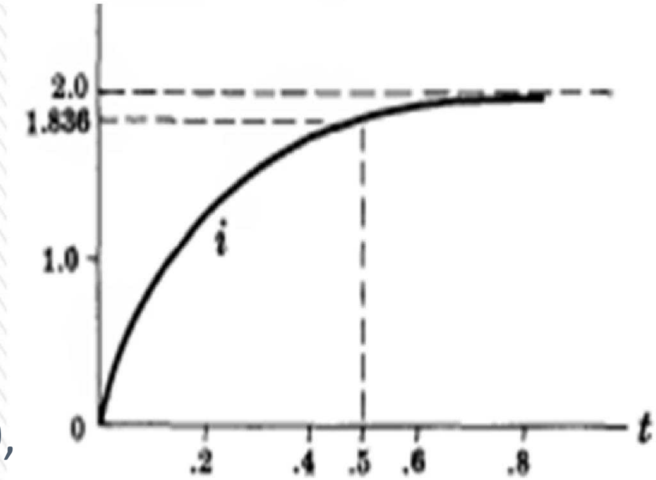
$$i = 2(1 - e^{-5t})$$

$$v_R = Ri = 100(1 - e^{-5t})$$

$$v_L = L\frac{di}{dt} = 100e^{-5t}$$

Examples

(b) Put $t = .5$ sec in (3) and obtain $i = 2(1 - e^{-5(.5)}) = 2(1 - .082) = 1.836$ amp.

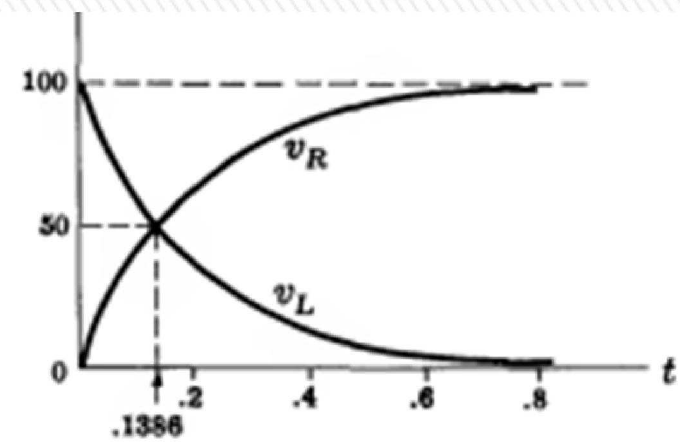


(b) For the two voltage to be equal:
each must be 50 volts since the applied voltage is 100,

$$v_L = 50 = 100e^{-5t}.$$

$$e^{-5t} = .5 \quad \text{or} \quad 5t = .693,$$

$$t = .1386 \text{ sec.}$$



Example (2)

In the series circuit shown in Fig.1 the switch is closed on position 1 at t = 0, thereby applying the 100 volt source to the RL branch, and at t = 500 μsec the switch is moved to position 2. Obtain the equations for the current in both intervals and sketch the transient.

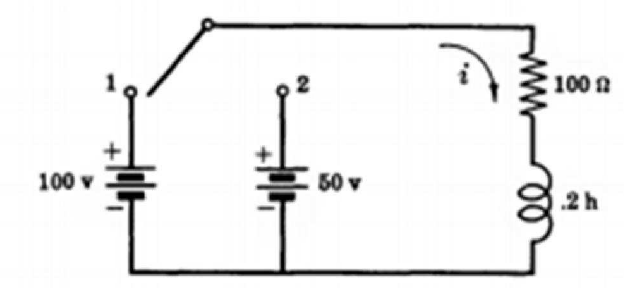
at Pos. 1

$$+100 = 100i + 0.2\frac{di}{dt}$$

$$500 = 500i + di/dt$$

$$(D + 500)i = 500$$

$$\boxed{i_{t_1} = 1 + Ae^{-500t}}$$



Cop. sol.

$D = 0 \quad i = 1$

P.I sol.

$0 = m + 500 \rightarrow m = -500$

$\therefore Ae^{-500t} + 1 = i_t$

at $t = 0 \quad i_0 = 0 \qquad A = -1$

$$\therefore \boxed{i_{t_1} = 1 - e^{-500t}} \longrightarrow \mathrm{I} \qquad \text{for } 0 < t < t_1$$

at $t = 500\ \mu\text{sec} \qquad i = 1 - e^{-500 \times 500\mu} = 0.221$ Amp

Example (2)

Switch Now at position $\underline{\underline{2}}$, $V_{source} = 50V$

$\therefore \quad 50 = 100\,i + 0.2\,di/dt \qquad$ or $(D + 500) = 250$

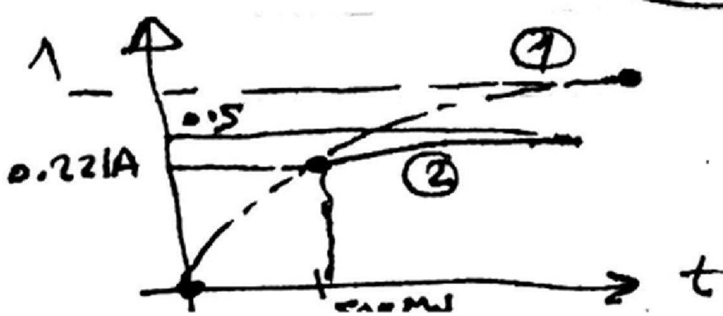
$i_{t_2} = 0.5 + Be^{-500\,t_2} \qquad$ where $t_2 = t - t_1$ ($t_1 \to 500\,ms$)

at $t = t_1 \quad \therefore t_2 = 0 \quad , i_2 = 0.221A$

$\therefore \quad 0.22A = 0.5 + B \quad \therefore \boxed{B = -0.279}$

for $t > t_1 \quad \therefore \quad \boxed{i_{t2} = 0.5 - 0.279\,e^{-500(t - t_1)}}$ ($t_1 \to 500\,ms$)

for $t > t_1$



Example (3)

A series RL circuit with R = 50 ohms and L = 0.2 H has a sinusoidal voltage source v = 150 sin (500t + Ø) applied at a time when Ø = 0. Find the complete current.

$$Ri + L\,di/dt = v$$

$$50i + 0.2\,di/dt = 150 \sin 500t$$

$$\therefore (D + 250)i = 750 \sin 500t$$

→ ~~complementry sol.~~

use Final equation $= c\,e^{-250t} + i_p$

$$i_p = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$= \frac{150}{\sqrt{(50)^2 + (500)^2 (0.2)^2}} \sin\left(500t + 0 - \tan^{-1}\left(\frac{500 \times 0.2}{50}\right)\right)$$

$$i_{total} = c\,e^{-250t} + 1.34 \sin(500t - 63.4^\circ)$$

at $t = 0$ $\quad i = 0$

$$\therefore 0 = c + 1.34 \sin(-63.4)$$

$$\therefore c = 1.2$$

$$\boxed{\therefore i_t = 1.2\,e^{-250t} + 1.34 \sin(500t - 63.4^\circ)}$$


Thank You